

Other Contributions

Mundell has made other contributions to macroeconomic theory. He has shown, for example, that higher inflation can induce investors to lower their cash balances in favor of increased real capital formation. As a result, even expected inflation might have a real economic effect – which has come to be known as *the Mundell-Tobin effect*. Mundell has also made lasting contributions to international trade theory. He has clarified how the international mobility of labor and capital tends to equalize commodity prices among countries, even if foreign trade is limited by trade barriers. This may be regarded as the mirror image of the well-known Heckscher-Ohlin-Samuelson result that free trade of goods tends to bring about equalization of the rewards to labor and capital among countries, even if international capital movements and migration are limited. These results provide a clear prediction: trade barriers stimulate international mobility of labor and capital, whereas barriers to migration and capital movements stimulate commodity trade.

3.20. John Forbes Nash (born 1928)



John F Nash's father, also called John Forbes Nash so we shall refer to him as John Nash Senior, was a native of Texas. John Nash Senior was born in 1892 and had an unhappy childhood from which he escaped when he studied electrical engineering at Texas Agricultural and Mechanical. After military service in France during World War I, John Nash Senior lectured on electrical engineering for a year at the University of Texas before joining the Appalachian Power Company in Bluefield, West Virginia. John F Nash's mother, Margaret Virginia Martin, was known as Virginia. She had a university education, studying languages at the Martha Washington College and then at West Virginia University. She was a school teacher for ten years before meeting John Nash Senior, and the two were married on 6 September 1924.

Johnny Nash, as he was called by his family, was born in Bluefield Sanatorium and baptised into the Episcopal Church. He was:

- ... *a singular little boy, solitary and introverted ...*

but he was brought up in a loving family surrounded by close relations who showed him much affection. After a couple of years Johnny had a sister when Martha was born. He seems to have shown a lot of interest in books when he was

young but little interest in playing with other children. His mother responded by enthusiastically encouraging Johnny's education, both by seeing that he got good schooling and also by teaching him herself.

Johnny's teachers at school certainly did not recognise his genius, and it would appear that he gave them little reason to realise that he had extraordinary talents. They were more conscious of his lack of social skills and, because of this, labelled him as backward. Although it is easy to be wise after the event, it now would appear that he was extremely bored at school. By the time he was about twelve years old he was showing great interest in carrying out scientific experiments in his room at home. It is fairly clear that he learnt more at home than he did at school.

Martha seems to have been a remarkably normal child while Johnny seemed different from other children. She wrote later in life:-

Johnny was always different. [My parents] knew he was different. And they knew he was bright. He always wanted to do things his way. Mother insisted I do things for him, that I include him in my friendships. ... but I wasn't too keen on showing off my somewhat odd brother.

Nash first showed an interest in mathematics when he was about 14 years old. Quite how he came to read E T Bell's *Men of mathematics* is unclear but certainly this book inspired him. He tried, and succeeded, in proving for himself results due to Fermat which Bell stated in his book. The excitement that Nash found here was in contrast to the mathematics that he studied at school which failed to interest him.

He entered Bluefield College in 1941 and there he took mathematics courses as well as science courses, in particular studying chemistry which was a favourite topic. He began to show abilities in mathematics, particularly in problem solving, but still with hardly any friends and behaving in a somewhat eccentric manner, this only added to his fellow pupils view of him as peculiar. He did not consider a career in mathematics at this time, however, which is not surprising since it was an unusual profession. Rather he assumed that he would study electrical engineering and follow his father but he continued to conduct his own chemistry experiments and was involved in making explosives which led to the death of one of his fellow pupils.

Nash won a scholarship in the George Westinghouse Competition and was accepted by the Carnegie Institute of Technology (now Carnegie-Mellon University) which he entered in June 1945 with the intention of taking a degree in chemical engineering. Soon, however, his growing interest in mathematics had him take courses on tensor calculus and relativity. There he came in contact with John Synge who had recently been appointed as Head of the Mathematics Department and taught the relativity course. Synge and the other mathematics professors

quickly recognised Nash's remarkable mathematical talents and persuaded him to become a mathematics specialist. They realised that he had the talent to become a professional mathematician and strongly encouraged him.

Nash quickly aspired to great things in mathematics. He took the William Lowell Putnam Mathematics Competition twice but, although he did well, he did not make the top five. It was a failure in Nash's eyes and one which he took badly. The Putnam Mathematics Competition was not the only thing going badly for Nash. Although his mathematics professors heaped praise on him, his fellow students found him a very strange person. Physically he was strong and this saved him from being bullied, but his fellow students took delight in making fun of Nash who they saw as an awkward immature person displaying childish tantrums. One of his fellow students wrote:

- We tormented poor John. We were very unkind. We were obnoxious. We sensed he had a mental problem.

Nash received a BA and an MA in mathematics in 1948. By this time he had been accepted into the mathematics programme at Harvard, Princeton, Chicago and Michigan. Now he felt that Harvard was the leading university and so he wanted to go there, but on the other hand their offer to him was less generous than that of Princeton. Nash felt that Princeton were keen that he went there while he felt that his lack of success in the Putnam Mathematics Competition meant that Harvard were less enthusiastic. He took a while to make his decision, while he was encouraged by Synge and his other professors to accept Princeton. When Lefschetz offered him the most prestigious Fellowship that Princeton had, Nash made his decision to study there.

In September 1948 Nash entered Princeton where he showed an interest in a broad range of pure mathematics: topology, algebraic geometry, game theory and logic were among his interests but he seems to have avoided attending lectures. Usually those who decide not to learn through lectures turn to books but this appears not to be so for Nash who decided not to learn mathematics "second-hand" but rather to develop topics himself. In many ways this approach was successful for it did contribute to him developing into one of the most original of mathematicians who would attack a problem in a totally novel way.

In 1949, while studying for his doctorate, he wrote a paper which 45 years later was to win a Nobel prize for economics. During this period Nash established the mathematical principles of game theory. P Ordeshook wrote:

- The concept of a Nash equilibrium n -tuple is perhaps the most important idea in noncooperative game theory. ... Whether we are analysing candidates' election strategies, the causes of war, agenda manipulation in legislatures, or the actions of interest groups, predictions about events reduce to a search for and description

of equilibrium. Put simply, equilibrium strategies are the things that we predict about people.

Milnor, who was a fellow student, describes Nash during his years at Princeton in:-

He was always full of mathematical ideas, not only on game theory, but in geometry and topology as well. However, my most vivid memory of this time is of the many games which were played in the common room. I was introduced to Go and Kriegspiel, and also to an ingenious topological game which we called Nash in honor of the inventor.

In fact the game “Nash” was almost identical to Hex which had been invented independently by Piet Hein in Denmark.

In 1950 Nash received his doctorate from Princeton with a thesis entitled *Non-cooperative Games*. In the summer of that year he worked for the RAND Corporation where his work on game theory made him a leading expert on the Cold War conflict which dominated RAND’s work. He worked there from time to time over the next few years as the Corporation tried to apply game theory to military and diplomatic strategy. Back at Princeton in the autumn of 1950 he began to work seriously on pure mathematical problems. It might seem that someone who had just introduced ideas which would, one day, be considered worthy of a Nobel Prize would have no problems finding an academic post. However, Nash’s work was not seen at the time to be of outstanding importance and he saw that he needed to make his mark in other ways. We should also note that it was not really a move towards pure mathematics for he had always considered himself a pure mathematician. He had already obtained results on manifolds and algebraic varieties before writing his thesis on game theory. His famous theorem, that any compact real manifold is diffeomorphic to a component of a real-algebraic variety, was thought of by Nash as a possible result to fall back on if his work on game theory was not considered suitable for a doctoral thesis.

In 1952 Nash published *Real algebraic manifolds* in the *Annals of Mathematics*. The most important result in this paper is that two real algebraic manifolds are equivalent if and only if they are analytically homeomorphic. Although publication of this paper on manifolds established him as a leading mathematician, not everyone at Princeton was prepared to see him join the Faculty there. This was nothing to do with his mathematical ability which everyone accepted as outstanding, but rather some mathematicians such as Artin felt that they could not have Nash as a colleague due to his aggressive personality.

From 1952 Nash taught at the Massachusetts Institute of Technology but his teaching was unusual (and unpopular with students) and his examining methods

were highly unorthodox. His research on the theory of real algebraic varieties, Riemannian geometry, parabolic and elliptic equations was, however, extremely deep and significant in the development of all these topics. His paper *C^1 isometric imbeddings* was published in 1954 and Chern, in a review, noted that it:

- ... contains some surprising results on the C^1 -isometric imbedding into an Euclidean space of a Riemannian manifold with a positive definite C^0 -metric.

Nash continued to develop this work in the paper *The imbedding problem for Riemannian manifolds* published in 1956. This paper contains his famous deep implicit function theorem. After this Nash worked on ideas that would appear in his paper *Continuity of solutions of parabolic and elliptic equations* which was published in the *American Journal of Mathematics* in 1958. Nash, however, was very disappointed when he discovered that E De Giorgi has proved similar results by completely different methods.

The outstanding results which Nash had obtained in the course of a few years put him into contention for a 1958 Fields' Medal but with his work on parabolic and elliptic equations was still unpublished when the Committee made their decisions he did not make it. One imagines that the Committee would have expected him to be a leading contender, perhaps even a virtual certainty, for a 1962 Fields' Medal but mental illness destroyed his career long before those decisions were made.

During his time at MIT Nash began to have personal problems with his life which were in addition to the social difficulties he had always suffered. He met Eleanor Stier and they had a son, John David Stier, who was born on 19 June 1953. Nash did not want to marry Eleanor although she tried hard to persuade him. In the summer of 1954, while working for RAND, Nash was arrested in a police operation to trap homosexuals. He was dismissed from RAND.

One of Nash's students at MIT, Alicia Larde, became friendly with him and by the summer of 1955 they were seeing each other regularly. In 1956 Nash's parents found out about his continuing affair with Eleanor and about his son John David Stier. The shock may have contributed to the death of Nash's father soon after but even if it did not Nash may have blamed himself. In February of 1957 Nash married Alicia; by the autumn of 1958 she was pregnant but, a couple of months later near the end of 1958, Nash's mental state became very disturbed.

Norbert Wiener was one of the first to recognize that Nash's extreme eccentricities and personality problems were actually symptoms of a medical disorder. A long sad episode followed which included periods of hospital treatment, temporary recovery, then further treatment. Alicia eventually divorced Nash, although she continued to try to help him, and after a period of extreme mental torture he appeared to become lost to the world, removed from ordinary society, although he

spent much of his time in the Mathematics Department at Princeton. The book is highly recommended for its moving account of Nash's mental sufferings.

Slowly over many years Nash recovered. He delivered a paper at the tenth World Congress of Psychiatry in 1996 describing his illness; it is reported in. He was described in 1958 as the:

- ... most promising young mathematician in the world ...

but he soon began to feel that:

- ... the staff at my university, the Massachusetts Institute of Technology, and later all of Boston were behaving strangely towards me. ... I started to see crypto-communists everywhere ... I started to think I was a man of great religious importance, and to hear voices all the time. I began to hear something like telephone calls in my head, from people opposed to my ideas. ... The delirium was like a dream from which I seemed never to awake.

Despite spending periods in hospital because of his mental condition, his mathematical work continued to have success after success. He said:

- I would not dare to say that there is a direct relation between mathematics and madness, but there is no doubt that great mathematicians suffer from manic characteristics, delirium and symptoms of schizophrenia.

In the 1990s Nash made a recovery from the schizophrenia from which he had suffered since 1959. His ability to produce mathematics of the highest quality did not totally leave him. He said:

- I would not treat myself as recovered if I could not produce good things in my work.

Nash was awarded (jointly with Harsanyi and Selten) the 1994 Nobel Prize in Economic Science for his work on game theory. In 1999 he was awarded the Leroy P Steele Prize by the American Mathematical Society:

- ... for a seminal contribution to research.

ECONOMISTS SHARE NOBELTRIO PIONEERED USE OF GAME THEORY IN THE FIELD

Three economists who were pioneers in using games like chess and poker as the foundation for understanding complex economic issues were awarded the Nobel Prize in economics yesterday -- exactly half a century after John Von Neumann and Oskar Morgenstern launched the field with the publication of "The Theory of Games and Economic Behavior."

John F. Nash of Princeton University, John C. Harsanyi of the University of California at Berkeley and Reinhard Selten of the Rheinische Friedrich- Wilhelms-Universitat in Bonn, will share the award, which this year amounts to \$930,000.

It marks the first time that the Swedes have recognized work in game theory.

The significance of Von Neumann and Morgenstern's contribution was recognized by economists and others almost immediately. The lessons they drew from homely games like chess and poker had nearly universal application to economic situations in which the participants had the power to anticipate and affect other participants' actions – that is, that there was a strategic aspect to most behavior.

But economists had little immediate success in applying their insights to a field whose preoccupation with the idea of “free competition” required that the ability of each particular participant to influence outcomes be negligible.

So instead, game theory found all kinds of immediate applications in the 1950s to problems of the Cold War, everything from airplane dog-fights to doctrines of massive retaliation. In a fascinating paperback book called “Prisoner's Dilemma,” writer William Poundstone records the heady intellectual excitement around the Institute for Advanced Study at Princeton and Rand Corp. in Santa Monica, Calif., where much of the early work was done.

Nash notched the first formal breakthrough while a young instructor at the Massachusetts Institute of Technology when he succeeded in generalizing a set of problems known to economists since the 1840s, when Augustine Cournot began writing about what might happen when two big companies collide with one another in the marketplace.

Nash formulated a universal “solution concept” for many-person “noncooperative” games (meaning those in which no outside authority assures that players stick to some predetermined rules). His name was thus attached to the whole range of possibilities that might arise from successfully seeing through a rival's strategy – they have been “Nash equilibria” ever since. “It was a very deep achievement,” said Princeton's Avinash Dixit, who was among those who nominated Nash for the prize.

Nash accomplished many other things, including introducing into economics a formal theory of bargaining (which the Swedes did not mention in the main body of their citation). But he made his way mainly as a pure mathematician, doing widely admired work, exhibiting many of the eccentricities that are associated with the caricature of that professional type.

Then in 1968, while on leave from Princeton in Paris, he suffered an extensive nervous breakdown, and has done no work in his field since.

Friends say that just in the last few months has Nash, now 66, resumed sitting with other mathematicians at meals and teas at the Institute for Advanced Study in Princeton, instead of keeping to himself. His hold on the larger world still seems

frail, they say, and it is unlikely that he will take full advantage of the spotlight that a trip to Stockholm affords to most laureates.

Once again, the Nobel award illuminates the Swedish angle of vision in economics, in which deep work on which rigorous foundations are erected is preferred to more intuitive and revealing styles. In doing so, they passed up the opportunity to honor the discoverers of the “prisoner’s dilemma,” a bargaining situation of wide applicability. They also ducked the chance to recognize Thomas Schelling, a University of Maryland economist who showed how many game theory concepts could be applied to economics. Hence the awards to Harsanyi, 74, a theorist who fled Hungary for the United States in 1956, and Selten, 64, a German scholar. Both researchers proved important mathematical theorems while refining the concept of Nash equilibrium, and Harsanyi in particular has ventured into topics of philosophy.

Neither is likely to turn up on the nightly news, however, arguing passionately for their views; their bliss has to do with highly abstract mathematical models of conflict and cooperation, of potential price wars and illegal collusion.